Three-Dimensional Thrust Nozzle Design for Maximum Axial Thrust

LYNN E. SNYDER* AND H. DOYLE THOMPSON†
Purdue University, Lafayette, Ind.

Theme

THREE-dimensional (nonaxisymmetric) supersonic nozzles may be required, e.g., for airbreathing propulsion systems where a high degree of integration of engine and vehicle structure is required. Each nozzle designed by the technique described herein produces the maximum axial thrust for a prescribed upstream flowfield, mass flow rate, exit lip shape and position, and ambient pressure. The technique is outlined, and results for a representative case are presented.

Content

This application of the calculus of variations to the design of three-dimensional (3-D) nozzles permits one to determine the maximum axial thrust which can be delivered by a nozzle with specific geometrical constraints, and to reduce costly analytical and experimental studies of effects of nozzle geometry on performance. The present formulation of the variational problem generally follows that of Thompson and Murthy[1] but possesses several important differences: no differential constraint is introduced into the fundamental function specifically to prevent the existence of Beltrami flow on the control surface (in the numerical cases presented here and in the full paper, an irrotational kernel is used to generate an irrotational flow on the control surface), the boundary conditions are imposed in a different manner, and the solution technique is unique.

The problem is formulated for the 3-D, supersonic, isoenergetic, homentropic flow of a perfect gas. The axial thrust is written as a surface integral over a control surface that passes through the exit lip and intersects the kernel region. The kernel region is that portion of the flowfield that can be determined from the prescribed initial conditions (see Figs. 1 and 2). The conditions in the kernel region, the mass flow rate, and the nozzle length are held fixed. The integral thrust expression is maximized by applying the calculus of variations to obtain two partial differential equations and three algebraic relations which relate the flow variables on an optimal control surface. Boundary conditions which apply along the exit lip and correspond to the length constraint are also obtained from the variational solution.

The uniqueness of the solution is established by proving that an optimal control surface is a characteristic surface, a result which assures a unique matching of the flow in the kernel with the flow across the control surface. (When the flow is constrained to be axisymmetric, the solution reduces to the well-known result obtained by Rao.[2]) The five design equations are solved numerically. The over-all solution procedure is such that an optimal nozzle is obtained for each set of initial conditions.

Once the optimal control surface has been determined, the nozzle contour can be found by computing the 3-D flowfield between the kernel and the control surface and then following the streamlines from the exit lip back to their origin. (This final part of the design procedure relies on the theory developed earlier.[3,4]) The exit contour, the exit flow properties, and the axial thrust can be determined and compared for whole families of optimal nozzles without having to complete the contour determination. In the present research the main emphasis is to establish: 1) that this formulation of the 3-D optimization problem is complete, 2) that the optimization procedure can be implemented, 3) that the resulting nozzles are significantly better than arbitrarily designed nozzles, and 4) the manner in which these nozzles differ from nonoptimally designed 3-D nozzles.

Results for nine nozzles are presented in the full paper and in Refs. 5 and 6. The two samples presented in Figs. 3 and 4 are typical of the others. The upper part of Fig. 3 shows the projections of the throat, the initial value line (the initial value line is the intersection of the control surface and the

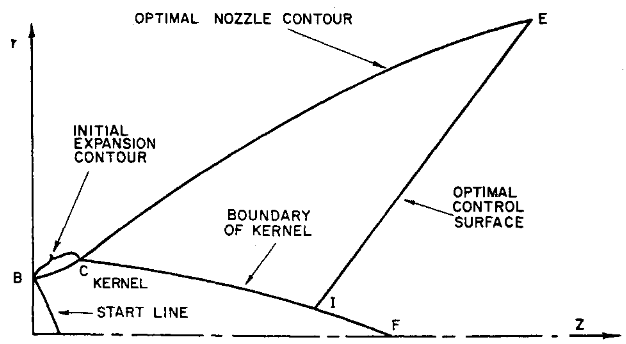


Fig. 1 Axisymmetric nozzle geometry.

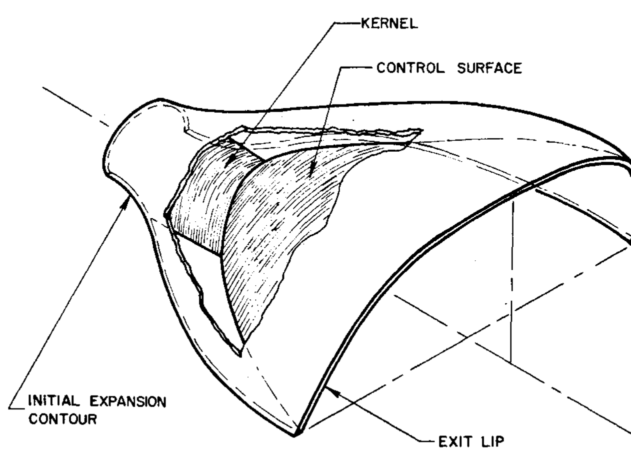


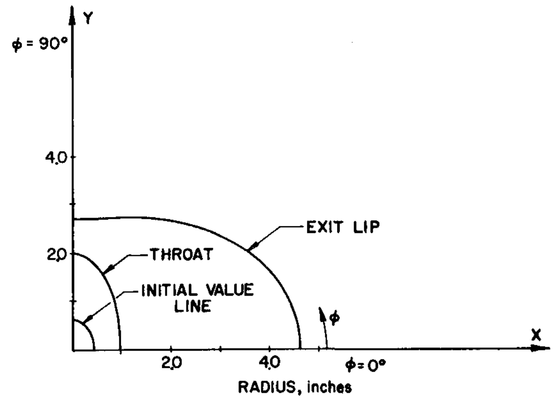
Fig. 2 Three-dimensional nozzle geometry.

Presented as Paper 71-42 at AIAA 9th Aerospace Sciences Meeting, New York, January 25–27, 1971; submitted Feb. 11, 1971; synoptic received April 21, 1971; revision received June 30, 1971. Full paper is available from AIAA. Price: AIAA members, $1.50; nonmembers, $2.00. Microfiche, $1.00. **Order must be accompanied by remittance.** This research was sponsored by AFAPL, Wright-Patterson Air Force Base, Ohio, Contract F33615-67-C-1068.

Index categories: Subsonic and Supersonic Airbreathing Propulsion; Hypersonic Airbreathing Propulsion; and Nozzle and Channel Flow.

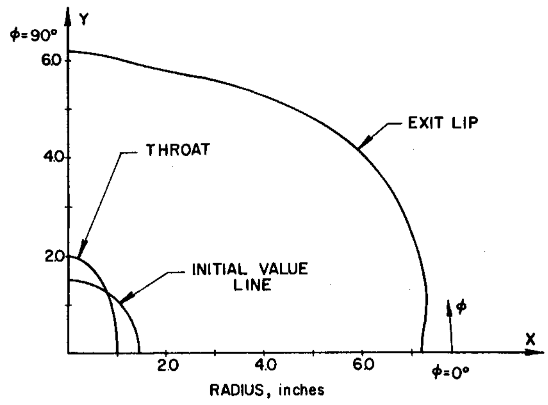
* Research Associate, School of Mechanical Engineering; presently Assistant Project Engineer, Pratt and Whitney Aircraft, E. Hartford, Conn. Associate Member AIAA.

† Associate Professor, School of Mechanical Engineering. Member AIAA.



ϕ	Z_{IVL}	θ_{IVL}	M_{IVL}	ψ_{IVL}	Z_e	θ_e	M_e	ψ_e
0.0°	4.20"	6.65°	2.82	0.00°	13.8"	2.50°	2.94	0.00°
15.0	4.20	6.62	2.82	- 3.85	13.3	2.50	2.94	0.52
30.0	4.21	6.52	2.82	- 8.94	12.3	2.49	2.94	0.80
45.0	4.21	6.30	2.83	-11.43	11.3	2.49	2.94	1.25
60.0	4.22	6.05	2.84	-10.26	10.1	2.50	2.94	0.46
75.0	4.24	5.76	2.84	- 3.61	9.4	2.50	2.94	-0.30
90.0	4.24	5.66	2.84	0.00	9.2	2.50	2.94	0.00

Fig. 3 Summary for optimal nozzle 1.



ϕ	Z_{IVL}	θ_{IVL}	M_{IVL}	ψ_{IVL}	Z_e	θ_e	M_e	ψ_e
0.0°	6.89"	12.74°	3.52	0.00°	19.6"	6.14°	3.65	0.00°
15.0	6.88	12.65	3.52	-2.16	20.2	6.17	3.65	0.10
30.0	6.93	12.62	3.54	-3.50	20.1	6.12	3.65	1.29
45.0	6.87	12.25	3.54	-4.22	19.1	6.12	3.65	1.45
60.0	6.82	11.90	3.53	-4.03	18.0	6.10	3.65	0.57
75.0	6.81	11.70	3.54	-1.89	17.1	6.14	3.65	-0.25
90.0	6.80	11.60	3.54	0.00	17.4	6.14	3.65	0.00

Fig. 4 Summary for optimal nozzle 2.

kernel, the locus of points I in Fig. 1); and the exit lip onto the r,ϕ-plane (the nozzle has two planes of symmetry, ϕ = 0° and 90°) for a low pressure ratio nozzle. The flow in the throat is parallel, uniform flow at Mach 1.05. An interesting result is that the exit lip has a generally elliptical projection on the r,ϕ-plane whose major axis is 90° to that of the elliptical throat.

The tabulation in Fig. 3 shows the values of axial position Z, flow angle θ, Mach number M, and flow angle ψ along the initial value line (subscript IVL) for $0° \leq \phi \leq 90°$, where

$$V_r = V \sin\phi \cos\psi; \quad V_\phi = V \sin\theta \sin\psi; \quad V_Z = V \cos\theta \quad (1)$$

and V is the magnitude of the velocity vector, and V_r, V_ϕ, and V_Z are the radial, tangential, and axial components of the velocity vector, respectively. The value of ψ indicates the magnitude of the three-dimensionality of the flow in the kernel.

Fig. 3 also shows properties along the exit lip (subscript e) of the optimal nozzle; the radius (see plot) and Z_e vary with ϕ. The values of ψ_{IVL} and ψ_e indicate that the flow on the control surface near the initial value lines has a cross flow component in a counterclockwise direction, whereas near the exit lip the cross flow component is in the clockwise direction. Another result is that this nozzle is optimally designed for a pressure ratio P_o/p_a of 49.7, where P_o and p_a are the stagnation and ambient pressures. With a design specific heat ratio γ of 1.2, its calculated inviscid thrust coefficient is 99.6% of the ideal value ($C_f/C_f{}^* = 0.996$).

To further evaluate the performance of this nozzle, several 3-D comparison nozzles having the same starting flow conditions and initial expansion contour but different exit lip shapes were designed and analyzed using the 3-D flow analysis program.[4] They were designed to $Z_e \simeq 13$ in. (invariant with ϕ) and ~5 in. radius, with super-elliptical cross-sections. At $P_o/p_a = 49.7$, these nozzles produced ~4% less axial thrust than the optimally designed nozzle.

Fig. 4 shows a quadrant and the initial-value-line and exit lip properties for a higher pressure ratio optimal nozzle having the same throat contour as nozzle 1, with design values γ = 1.2, $P_o/p_a = 314$, and $C_f/C_f{}^* = 0.993$.

References

[1] Thompson, H. D. and Murthy, S. N. B., "Design of Optimized Three-Dimensional Nozzles," *Journal of Spacecraft and Rockets*, Vol. 3, No. 9, Sept. 1966, pp. 1384–1393.

[2] Rao, G. V. R., "Exhaust Nozzle Contour for Optimum Thrust," *Jet Propulsion*, Vol. 28, 1958, pp. 377–382.

[3] Ransom, V. H., Hoffman, J. D., and Thompson, H. D., "A Second-Order Numerical Method of Characteristics for Three-Dimensional Supersonic Flow, Volume I, Theoretical Development and Results," AFAPL-TR-69-98, Oct. 1969, Air Force Aero Propulsion Lab., Wright-Patterson Air Force Base, Ohio.

[4] Ranson, V. H., Cline, M. C., Hoffman, J. D., and Thompson, H. D., "Volume II, Computer Program Manual," AFAPL-TR-69-68, Jan. 1970, Air Force Aero Propulsion Lab., Wright-Patterson Air Force Base, Ohio.

[5] Snyder, Lynn E., "Three-Dimensional Nozzle Design for Maximum Thrust," Ph.D. thesis, Purdue University, Jan. 1971.

[6] Snyder, Lynn E. and Thompson, H. Doyle, "Three-Dimensional Nozzle Design for Maximum Thrust, Volume I, Theoretical Development and Results," AFAPL-TR-70-77, Oct. 1970; ("Volume II, Computer Program Manual," is in preparation), Air Force Aero Propulsion Lab., Wright-Patterson Air Force Base, Ohio.